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## The Term Structure of Uncertainty: New Evidence from Survey Expectations

We construct measures of forecasters' subjective uncertainty at horizons from 1 to 5 years, using the European Central Bank's Survey of Professional Forecasters. The uncertainty curve is more linear than the disagreement curve. We document heterogeneity across forecasters in the level and the term structure of uncertainty, and show that the difference between long-run and short-run uncertainty is procyclical. We develop a signal extraction model that features (i) Kalman filter updating, (ii) time-varying uncertainty, and (iii) assessment of multistep ahead uncertainty. Heterogeneous patterns of uncertainty over different horizons depend on perceived persistence and variability of the signal and the noise.

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HEIGHTENED UNCERTAINTY IN THE GREAT RECESSION has prompted renewed efforts to investigate the sources and consequences of uncertainty (Bloom 2009, Leduc and Liu 2016, Kozeniauskas, Orlik, and Veldkamp 2018). Survey forecasts provide valuable information about the expectation formation process and the associated subjective uncertainty (Coibion and Gorodnichenko 2012, Ben-David, Graham, and Harvey 2013).

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In this paper, we construct measures of individual forecasters' subjective uncertainty at horizons ranging from 1 to 5 years, using density (histogram) forecasts from the European Central Bank's Survey of Professional Forecasters (ECB SPF). Uncertainty refers to the spread (e.g., variance) of an individual agent's probability distribution about an outcome. We also construct measures of disagreement, or dispersion of expectations across forecasters, at each horizon. We explore the properties of uncertainty and disagreement over shorter and longer horizons, documenting four stylized facts.

First, the term structure of uncertainty is highly *linear*—that is, uncertainty at the 1- and 2-year horizons can almost perfectly predict uncertainty at the 5-year horizon. This is true for both aggregate uncertainty and at the individual forecaster level.

Second, the slope of the term structure of uncertainty is time-varying. We confirm that uncertainty is countercyclical, but also show that the slope is *procyclical*. In recessions, short-run uncertainty rises significantly more than long-run uncertainty.

Third, forecasters are overconfident at all horizons, in the sense that *ex post* uncertainty is higher than *ex ante* uncertainty. For unemployment, the forecasters are most overconfident at the longest horizon.

Fourth, we document substantial heterogeneity across forecasters in both the level and term structure of uncertainty. While average uncertainty increases with forecast horizon, a sizeable minority of forecasters have higher uncertainty at shorter horizons. This heterogeneity is persistent. That is, particular forecasters tend to have particularly wide or narrow (or even inverted) term structures of uncertainty.

Guided by our stylized facts, we model forecasters' signal extraction process under an information structure with private and public channels of information. We adopt the framework of Baker, McElroy, and Sheng (2020) that features Kalman filter updating and state-dependent information processing. We generalize their framework by allowing for time-varying uncertainty in both the signal and the noise and by studying uncertainty and disagreement across multiple forecast horizons. The interplay between signal and noise that is metrized through signal-to-noise ratio (SNR) plays an essential role in establishing the last three stylized facts. The sticky information model *à la* Mankiw and Reis (2002) cannot explain the linear term structure of uncertainty. Classical noisy information model *à la* Sims (2003) can account for this linearity, but cannot explain the procyclical term structure of uncertainty. While a basic VAR model alone can also explain this linearity, it cannot account for the other empirically observed phenomena.

Our theory emphasizes that perceived persistence is the key to understanding multi-step ahead expectation formation. When the signal is perceived as being more persistent, the term structure of uncertainty takes on an increasingly linear pattern. For the less persistent signal, uncertainty increases at all horizons, but the increase is sharper at shorter horizons, leading to a procyclical term structure. This result supports

intrinsic expectations persistence in Fuhrer (2017, 2018).<sup>1</sup> Perceived variability also plays an important role. When the perceived noise variability is substantially lower than the actual noise variability, agents lower their forecast uncertainty uniformly across horizons, resulting in overconfidence. In contrast, when the signal variability is perceived by some agents to be higher at shorter horizons, but lower at longer horizons, the regular ordering of uncertainty might be inverted across horizons for these agents.

Our paper contributes to several strands of a broad literature using survey data to study expectations formation, information frictions, and uncertainty (Mankiw, Reis, and Wolfers 2004, Armantier et al. 2015, Coibion and Gorodnichenko 2015, Abel et al. 2016, Kozeniasukas, Orlik, and Veldkamp 2018). A subset of this literature makes use of the multiple-horizon forecasts that are available from some surveys to extract additional information (Andrade et al. 2016, Binder 2018). For example, Aruoba (2020) combines inflation forecasts at various horizons from several surveys to obtain a term structure of inflation expectations, and combines this with nominal interest data to obtain a term structure of *ex ante* real interest rates.

Several papers examine disagreement at various forecast horizons. Lahiri and Sheng (2008) use multihorizon data to estimate the relative importance of three components of disagreement: (i) differences in prior beliefs, (ii) different weights attached on priors, and (iii) differential interpretation of public information. In a similar vein, Patton and Timmermann (2010) show that the term structure of disagreement can be used to determine the relative importance of differences in priors versus differences in private information. Andrade and Le-Bihan (2013) emphasize two sources of heterogeneity: inattention and noisy signals, while Giacomini, Skreta, and Turén (2020) find that in normal times heterogeneous priors and inattention are enough to generate persistent disagreement, but not during the crisis.

Other papers examine uncertainty at multiple horizons. Inflation uncertainty at different horizons is of particular interest to monetary policymakers. Ball and Cecchetti (1990) find that the level of inflation has a stronger effect on the variance of permanent than of temporary shocks, and therefore has a greater effect on longer horizon than on shorter-horizon inflation uncertainty. These authors do not use a direct measure of inflation uncertainty, but rather use inflation variability as a proxy. Using data from the Michigan Survey of Consumers, Binder (2017) constructs an index of consumer inflation uncertainty and documents that the uncertainty was higher for the longer than shorter horizon until the mid-1990s. Since then, longer-run inflation uncertainty declined more than shorter-run uncertainty, inverting the term structure.

Barrero, Bloom, and Wright (2017) study uncertainty at short and long horizons using firm and macro implied volatility as proxies for uncertainty. As with our uncertainty measures, the term structure of implied volatility is linear, so they focus on just the 30-day and 1-year horizons as proxies for short-run and long-run

1. Fuhrer (2017) shows that intrinsic persistence in expectations, rather than price indexation or habit formation, is a key source of macro-economic persistence. Fuhrer (2018) further explores how expectations might exhibit such inertia and finds that agents smooth their expectations' response to news.

uncertainty. They show that long-run uncertainty is more strongly associated with economic policy uncertainty, and that R&D is relatively more sensitive to long-run uncertainty than investment. Berger, Dew-Becker, and Giglio (2020) study whether realized volatility or implied volatility are associated with contractionary movements in macro variables, finding that implied volatility has difficulty in explaining contractions after controlling for realized volatility. Clark, McCracken, and Mertens (2020) estimate uncertainty from survey forecast errors at multiple horizons and find that uncertainty measures move together strongly across forecast horizons. Breitung and Knüppel (2018) examine forecast error variance across horizons using Consensus Economics survey data, and show that forecast error variance at longer horizons is often as large as the unconditional variance of the target variable, indicating that these longer-horizon forecasts are minimally informative.

We differ from these papers in the measures of uncertainty and the horizons that we examine, and in simultaneously examining the term structures of both uncertainty and disagreement. Instead of implied volatility for firms or various proxies for uncertainty, such as *ex post* accuracy, we directly measure forecasters' *ex ante* subjective uncertainty about three key macro-economic variables, at horizons up to 5 years. A benefit of looking at these longer horizons is that they correspond to the horizons over which some macro-economic policies may take effect, and also the horizons that firms may use when evaluating investment projects. Taking advantage of the panel data structure, we examine and model the uncertainty, disagreement, and heterogeneity in the term structure of uncertainty for forecasters who are making forecasts of the same variables over time.

The paper proceeds as follows. Section 1 describes the ECB's density forecast data set. We establish four new stylized facts about forecast uncertainty at varying horizons in Section 2. In Section 3, we propose a theory of expectation updating and illustrate the model implications through simulations. Section 4 concludes. Additional estimation results and technical proofs are relegated to an appendix.

## 1. DATA AND MEASUREMENT OF UNCERTAINTY

The ECB has conducted the Survey of Professional Forecasters (SPF) since 1999. Each quarter, approximately 60 forecasters provide point and density forecasts for inflation ( $\pi$ ), GDP growth ( $g$ ), and unemployment ( $u$ ) over several horizons. Density forecasts take the form of histograms; the forecaster assigns probabilities, summing to 100%, that the realization will fall in each bin.<sup>2</sup> The density forecasts have a consistent

2. Less than half a percent of density forecasts assign 100% probability to a single bin. On other surveys where respondents are asked to provide the probability of an event, there is often a "seemingly inappropriate blip" at 50% (Fischhoff and de Bruin 1999). This does not appear to be a problem in the SPF density forecasts, for which less than 6% of assigned nonzero bin probabilities are 50%. Just under half of the assigned nonzero bin probabilities are multiples of 10 percentage points, and around a quarter are other multiples of 5 percentage points.

bin width of 0.5 percentage points. This is an advantage over the U.S. SPF, for which the bin width changes over time. The number of bins occasionally changes to try to ensure that large probabilities are not assigned to the upper and lower intervals, which are open-ended. Thus, for example, additional lower bins were added for the inflation and growth density forecasts in 2009.<sup>3</sup>

We consider forecasts at three horizons that are included on most survey dates: 1-year, 2-year, and 5-year. The 1- and 2-year forecasts have fixed horizons, where the fixed 1-year horizon refers to the month (for inflation and unemployment) or quarter (for growth) 1 year ahead of the latest available observation at the time of the survey. The fixed 2-year horizon refers to the month or quarter 2 years ahead of the latest available observation. The 5-year forecasts, unlike the 1- and 2-year forecasts, are fixed event forecasts. Specifically, for surveys on the third and fourth quarter of the year, the 5-year horizon refers to five calendar years ahead, while on the first and second quarter of the year, the 5-year horizon refers to four calendar years ahead. Given the long time horizon, we assume that these forecasts approximate a 5-year fixed horizon forecast of the same variable.

### 1.1 Uncertainty Measurement

We use the density forecasts to construct a measure  $U_{ih}^x$  of *ex ante*, subjective uncertainty for each forecaster  $i$ , variable  $x$ , and horizon  $h$  in quarter  $t$ , defined as the variance of the density forecast. We estimate the variance and the mean of each forecaster's density forecasts parametrically.<sup>4</sup> We fit (via maximum likelihood) a generalized beta distribution to the density forecast, with supports determined by the individual forecast values. Liu and Sheng (2019) show that this distributional setting, which is highly flexible, performs best in terms of goodness of fit in mimicking the empirical histograms in the data, which can be asymmetric or irregular.<sup>5</sup> We also consider aggregate uncertainty  $\bar{U}_{ih}^x$ , the average across forecasters of  $U_{ih}^x$ .

3. In the first quarter of 2009, many forecasters assigned high probability—sometimes even 100%—to the lowest bin for growth, corresponding to growth rates less than  $-1\%$ . As a result, measured uncertainty in this period is artificially low, so we omit 2009Q1 growth uncertainty from subsequent analysis. In the second quarter, bins for growth rates of less than  $-6\%$ ,  $-6\%$  to  $-5.5\%$ , ...,  $-1.5\%$  to  $-1\%$  were added.

4. As Engelberg, Manski, and Williams (2009) discuss, the parametric approach imposes assumptions about the shapes of forecasters' subjective distributions—in particular, that they are unimodal—but enables sharper empirical analysis. The assumption of unimodality is not problematic in our setting. Over 85% of density forecasts have a single bin with maximum probability. In virtually all of the remaining forecasts with multiple modal bins, the multiple modal bins are adjacent. Thus, we see no evidence of multimodal distributions. All of our results are robust to using a nonparametric approach, which assumes that the probability is concentrated at the midpoint of each bin. The correlation coefficients between the parametric and nonparametric uncertainty estimates are above 0.99. The levels are similar, with the parametric uncertainty estimates slightly higher.

5. The four distribution settings that Liu and Sheng compare include the normal distribution, as used by Giordani and Soderlind (2003), the generalized beta distribution with no parameter constraint, the generalized beta distribution with supports determined by individual forecast values, and a combination of beta and triangle distributions, as used by Engelberg, Manski, and Williams (2009).

### 1.2 Measuring Disagreement and Errors

Let  $F_{it}^x$  denote forecaster  $i$ 's point forecast of variable  $x$  for horizon  $h$  made at time  $t$ , and let  $\mu_{it}^x$  denote the mean of her density forecast. Engelberg, Manski, and Williams (2009) show that for the U.S. SPF, point forecasts are usually close to the central tendency of density forecasts. Likewise, we find that the correlation between  $F_{it}^x$  and  $\mu_{it}^x$  is around 0.9 with respondents' point forecasts. See Figure A1 for plots of  $F_{it}^x$  against  $\mu_{it}^x$  for each variable and horizon.

Disagreement  $D_{it}^x$  is defined as the cross-sectional variance of forecasts for variable  $x$  at horizon  $h$ . Note that this could refer to the heterogeneity of point forecasts or of the density means across forecasters. Since both measures are very similar (with correlation coefficients around 0.8 or 0.9 depending on variable and horizon), for consistency with the literature we use the variance of  $F_{it}^x$ .

We also use the point forecast in defining a respondent's mean squared error (MSE). Data on the realization of the variables being forecasted is from Eurostat. More detailed information on these variables appears in Tables A1 and A2. The forecast error  $e_{it}^x$  is the difference between the actual realization and forecast. The MSE is sometimes used as a measure of *ex post* uncertainty or accuracy.

## 2. STYLIZED FACTS

In this section, we present four stylized facts from our empirical analysis of the term structures of uncertainty and disagreement.

- (1) *The term structure of uncertainty in the ECB SPF is highly linear—in particular, more linear than the term structure of disagreement.*

Figure 1 displays estimates of aggregate uncertainty by horizon for inflation, growth, and unemployment. For all variables, aggregate uncertainty increases with forecast horizon. *A priori*, this need not be the case. For example, if the central bank is highly credible and long-run inflation expectations are firmly anchored at the inflation target, then inflation uncertainty could decrease with forecast horizon (Beechey, Johannsen, and Levin 2011). Indeed, Binder (2017) shows that respondents to the Michigan Survey of Consumers had similar long-run and short-run inflation uncertainty until the late 1980s, and that since then, long-horizon uncertainty is lower than short-horizon uncertainty.

A term structure is *linear* if the entire curve is well characterized by a level and slope statistic. We regress aggregate 5-year uncertainty on 1-year uncertainty (the level) and the difference between 1- and 2-year uncertainty (the slope). Table 1 shows that aggregate uncertainty at the 5-year horizon is largely explained by the level and slope statistic, with  $R^2$  around 0.9 for each variable.<sup>6</sup> Table 2 shows a panel version

6. Regression coefficients are similar pre- and post-2008; the  $R^2$  values are slightly lower with shorter time samples, but still indicative of high linearity. See Table A3. Regression coefficients and  $R^2$  values are

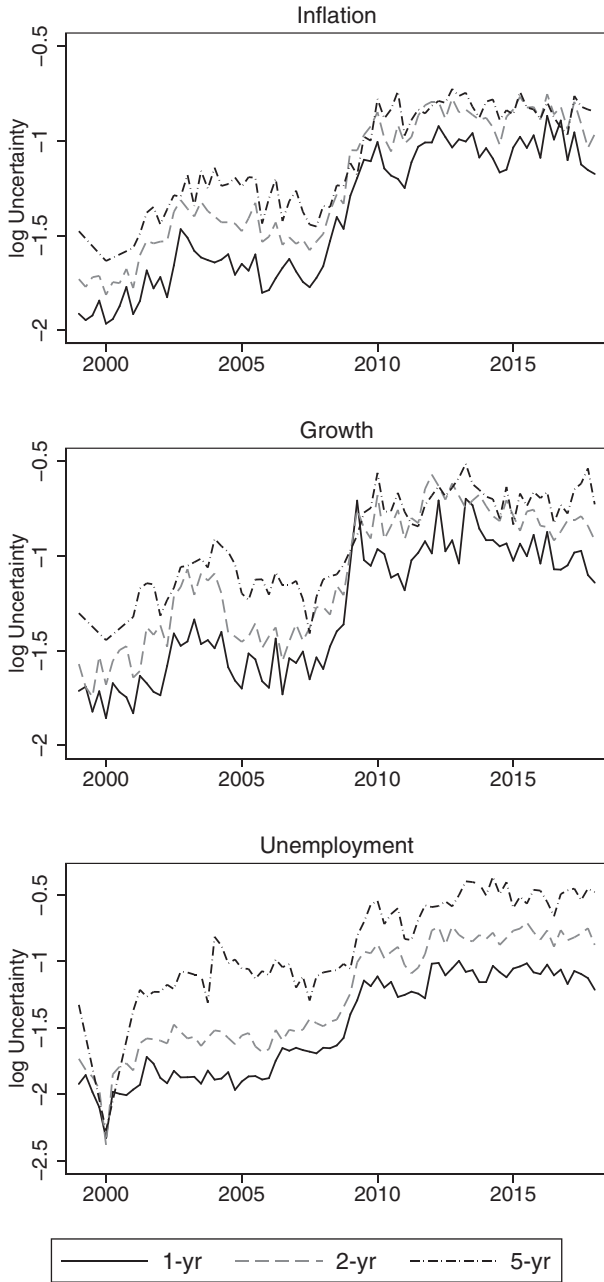


Fig 1. Time Series of Aggregate Uncertainty.

Notes: ECB SPF data. Uncertainty is the variance of a beta distribution fitted to an individual's density forecast. We take the average of log uncertainty across forecasters at time  $t$ .

TABLE 1  
REGRESSION FOR LINEARITY OF THE TERM STRUCTURE OF AGGREGATE UNCERTAINTY

	(1) Inflation	(2) Growth	(3) Unemp
1-year	0.80*** (0.02)	0.73*** (0.03)	0.79*** (0.04)
2-yr minus 1-yr	0.98*** (0.16)	0.46*** (0.12)	1.46*** (0.27)
Constant	-0.17*** (0.04)	-0.09* (0.05)	-0.06 (0.08)
Observations	71	70	71
$R^2$	0.93	0.88	0.90

NOTES: Robust, time-clustered standard errors in parentheses. Dependent variable is log uncertainty at 5-year horizon for indicated variable. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ .

TABLE 2  
PANEL REGRESSION FOR LINEARITY OF THE TERM STRUCTURE OF UNCERTAINTY

	(1) Inflation	(2) Growth	(3) Unemp	(4) Inflation	(5) Growth	(6) Unemp
1-year	0.84*** (0.04)	0.86*** (0.03)	0.80*** (0.04)	0.82*** (0.02)	0.85*** (0.02)	0.72*** (0.02)
2-yr minus 1-yr	0.76*** (0.04)	0.71*** (0.05)	0.77*** (0.05)	0.73*** (0.03)	0.69*** (0.03)	0.69*** (0.03)
Constant	-0.08 (0.05)	0.01 (0.04)	0.10 (0.07)	0.03 (0.07)	0.14** (0.07)	0.16* (0.09)
Observations	2,483	2,395	2,214	2,483	2,395	2,214
$R^2$	0.76	0.76	0.65	0.76	0.77	0.66
Time FE	No	No	No	Yes	Yes	Yes
Forecaster FE	No	No	No	Yes	Yes	Yes

NOTES: Robust, time-clustered standard errors in parentheses. Dependent variable is log uncertainty for forecaster  $i$  at 5-year horizon for indicated variable. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ .

of this regression, in which the dependent variable is 5-year uncertainty for forecaster  $i$  in quarter  $t$ . Columns (1)–(3) do not include time or forecaster fixed effects, while (4)–(6) do. Again, for all variables, the term structure of uncertainty is highly linear. In the similar spirit, Barrero, Bloom, and Wright (2017) document that volatility curves are linear, or in other words, that a regression of long-run implied volatility on short-run volatility and the difference between short- and medium-run volatility has a high  $R^2$ .<sup>7</sup>

also similar if uncertainty is measured using the interquartile range of the density forecast as in Abel et al. (2016), rather than the log variance.

7. Clements and Galvao (2017) study fixed-event forecasts in the U.S. SPF with forecasting horizons ranging from 8- to 1-quarter ahead and focus on aggregate uncertainty averaged both across forecasters and over time. In contrast, we analyze fixed-horizon forecasts in ECB SPF with horizons of 1-, 2- and 5-year ahead and study the cyclical nature of the term structure, as discussed in the second stylized fact.



The linearity of the uncertainty term structure is one feature that distinguishes uncertainty from disagreement. Other differences between uncertainty and disagreement are well-documented; for example, see Abel et al. (2016) and Rossi, Sekhposyan, and Soupre (2016), among others. As shown in Figure 2, disagreement also spikes in the Great Recession, but unlike uncertainty, does not remain elevated. For unemployment, disagreement increases monotonically with horizon, but the term structure is often inverted for the other variables. Analogous regressions to those in Table 1 for disagreement have  $R^2$  values of 0.39, 0.14, and 0.40 for inflation, growth, and unemployment, respectively.<sup>8</sup>

(2) *The difference between long-run and short-run uncertainty is procyclical.*

Both the level and term structure of uncertainty display some notable time variations. Uncertainty is countercyclical, but the slope of the term structure is procyclical. That is, in periods of low growth, uncertainty rises, but more so at the shorter horizon, so the term structure narrows. The level of uncertainty also rises, and the term structure narrows, when inflation is far from target. Uncertainty rises when there is high economic policy uncertainty (as measured by the Economic Policy Uncertainty Index for Europe from Baker, Bloom, and Davis 2016),<sup>9</sup> and the slope is unchanged or falls.

These patterns appear in the panel regressions with forecaster fixed effects in Table 3. For inflation, growth, and unemployment, we regress log uncertainty at the short and long horizon on growth, the deviation of inflation from target, the EPU, and forecaster fixed effects. We also regress the difference between 5- and 1-year log uncertainty and a dummy variable indicating that 5-year uncertainty is greater than 1-year uncertainty, on the same variables. For all variables, uncertainty declines with growth and increases with  $|\pi - 2|$ , and both effects are greater at the shorter horizon. Uncertainty increases with the EPU, with similar effects at each horizon. Column 4 shows that the slope decreases with  $|\pi - 2|$ , and for inflation and growth forecasts, the slope increases with  $g$ .

For inflation and growth, more than for unemployment, the difference between 5- and 1-year log uncertainty is strongly procyclical (see Figure 3). In particular, the lowest growth rates in the first two quarters of 2009 correspond to the narrowest term structure of growth uncertainty and the only time when aggregate 1-year growth uncertainty was greater than aggregate 5-year growth uncertainty. These results clearly

8. If disagreement is instead measured as the interquartile range of forecasts across forecasters, these  $R^2$  values are 0.06, 0.29, and 0.13, again indicating low linearity. The term structure of inflation and growth disagreement also frequently inverts with the interquartile range measure.

9. The EPU is based on newspaper articles regarding policy uncertainty. Data were downloaded in April 2018 from [http://www.policyuncertainty.com/europe\\_monthly.html](http://www.policyuncertainty.com/europe_monthly.html). The newspapers include Le Monde and Le Figaro for France, Handelsblatt and Frankfurter Allgemeine Zeitung for Germany, Corriere Della Sera and La Repubblica for Italy, El Mundo and El Pais for Spain, and The Times of London and Financial Times for the United Kingdom.

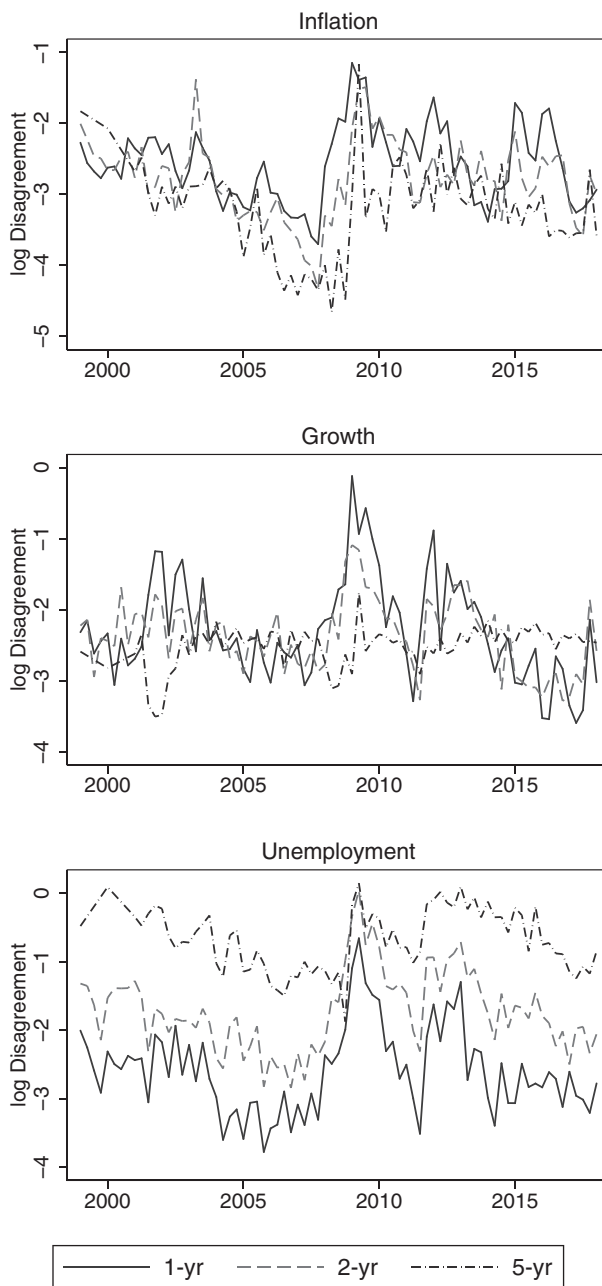


Fig 2. Time Series of Forecast Disagreement.

Notes: ECB SPF data. Disagreement is the cross-sectional variance of the point forecasts.

TABLE 3  
REGRESSIONS OF LEVEL AND SLOPE OF UNCERTAINTY ON MACRO VARIABLES

	(1) 1y	(2) 5y	(3) 5y-1y	(4) 5y > 1y
<i>A. Inflation uncertainty</i>				
$g$	-0.029*** (0.005)	-0.004 (0.006)	0.020*** (0.005)	0.012*** (0.005)
$ \pi - 2 $	0.119*** (0.015)	0.073*** (0.018)	-0.037** (0.015)	-0.041*** (0.013)
EPU/100	0.323*** (0.017)	0.246*** (0.020)	-0.037** (0.017)	0.001 (0.015)
$N$	3,424	2,777	2,571	2,571
$R^2$	0.17	0.07	0.02	0.01
<i>B. Growth uncertainty</i>				
$g$	-0.055*** (0.006)	-0.017** (0.007)	0.038*** (0.006)	0.026*** (0.005)
$ \pi - 2 $	0.099*** (0.016)	0.051*** (0.019)	-0.038** (0.015)	0.003 (0.013)
EPU/100	0.237*** (0.018)	0.199*** (0.021)	0.013 (0.018)	0.024 (0.015)
$N$	3,422	2,682	2,480	2,480
$R^2$	0.12	0.05	0.03	0.01
<i>C. Unemployment uncertainty</i>				
$g$	-0.017*** (0.006)	-0.007 (0.007)	0.004 (0.007)	-0.001 (0.004)
$ \pi - 2 $	0.163*** (0.018)	0.130*** (0.021)	-0.047** (0.020)	0.004 (0.012)
EPU/100	0.352*** (0.020)	0.300*** (0.024)	-0.018 (0.022)	-0.051*** (0.014)
$N$	3,085	2,501	2,256	2,256
$R^2$	0.16	0.09	0.00	0.01

NOTES: Robust, standard errors in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ . Panel regressions with forecaster fixed effects. In columns (1) and (2), dependent variable is log uncertainty for the 1- and 5-year horizon, respectively, for inflation, growth, or unemployment, as indicated. In column (3), dependent variable is the difference between 5- and 1-year log uncertainty. In column (4), dependent variable is a dummy variable indicating greater uncertainty at 5-year than at 1-year horizon. All regressions include a constant term. EPU is rescaled (divided by 100) for ease of presenting coefficients.

show that while the level of term structure of uncertainty is countercyclical, its slope is procyclical.

(3) *Forecasters are overconfident at all horizons.*

For each density forecast, we use the estimated parameters of the beta distribution to compute the 2.5th and 97.5th percentiles of the probability distribution. Table 4 shows that for each variable and horizon, substantially less than 95% of realizations are in this 95% confidence interval. Pooling across variables and horizons, 53% of realizations are within the 95% confidence interval.<sup>10</sup> This is consistent with evidence of overconfidence as previously documented for the U.S. SPF (Giordani and Soder-

10. Similarly, we also compute 70% confidence intervals and also find that for each variable and horizon, substantially less than 70% of realizations are within the 70% confidence interval. Pooling across variables and horizons, 36% of realizations are within the 70% confidence interval.

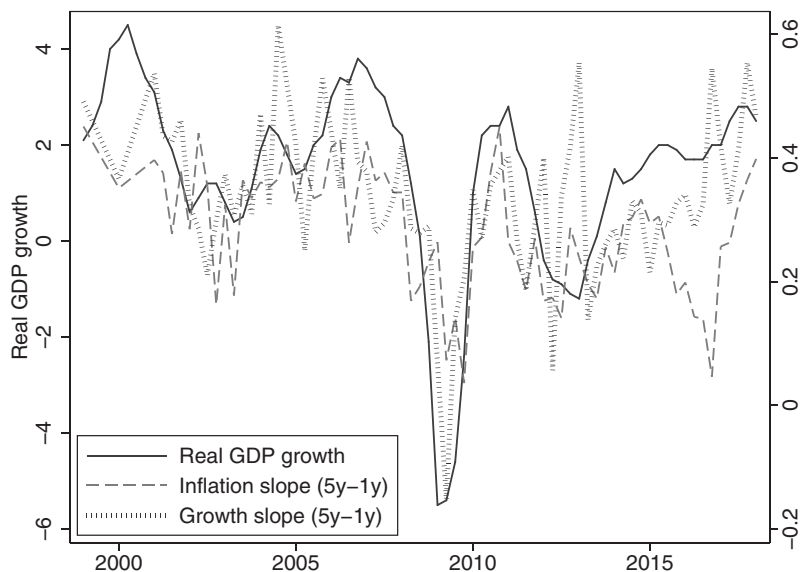


Fig 3. Economic Growth and Slope of Term Structure of Uncertainty.

NOTES: The slope of inflation uncertainty is  $\ln U_5^\pi - \ln U_1^\pi$ . The slope of growth uncertainty is  $\ln U_5^g - \ln U_1^g$ . Real GDP growth is the growth rate over the past year.

TABLE 4

FORECASTER OVERCONFIDENCE: SHARE OF REALIZATIONS IN 95% CONFIDENCE INTERVAL

Variable	Within 95% interval	Below 2.5th percentile	Above 97.5th percentile
$\pi$ , 1-yr	0.65	0.15	0.20
$\pi$ , 2-yr	0.64	0.21	0.15
$\pi$ , 5-yr	0.56	0.32	0.10
$g$ , 1-yr	0.44	0.30	0.27
$g$ , 2-yr	0.51	0.33	0.15
$g$ , 5-yr	0.61	0.32	0.07
$u$ , 1-yr	0.58	0.19	0.23
$u$ , 2-yr	0.43	0.23	0.34
$u$ , 5-yr	0.37	0.05	0.58
Pooled	0.53	0.23	0.23

NOTES: The table shows the share of realizations within the 95% confidence interval for each variable and horizon and for all variables and horizons pooled. Sample includes the 40 forecasters who make at least 25 density forecasts for each variable at each horizon.

lind 2003), U.S. chief financial officers (Ben-David, Graham, and Harvey 2013), and in experimental research in psychology (Rabin 1998).

Table 4 also shows the share of realizations above the 97.5 percentile and below the 2.5 percentile for each variable and horizon. Patterns of overconfidence by horizon differ by variable. For inflation, overconfidence is similar at each horizon. For growth, overconfidence is somewhat higher at the shorter horizon. The clearest

pattern is for unemployment, for which overconfidence is highest at the longest horizon. For growth and unemployment, forecasters often erred on the optimistic side: growth realizations were more frequently below the 2.5 percentile and unemployment realizations were more frequently above the 97.5 percentile (for inflation there is more symmetry).<sup>11</sup> This may reflect the unexpected duration and severity of the Great Recession. Overall overconfidence, however, was similar pre- and post-2008.

Forecaster overconfidence means that *ex ante* and *ex post* uncertainty differ substantially. Figure A2 plots mean squared forecast errors for each variable and horizon. The *ex post* errors are *larger* than would be implied by the subjective variances of the density forecasts. Abel et al. (2016), also using the ECB-SPF data, find very little correlation between *ex ante* uncertainty and *ex post* accuracy.

(4) *There is substantial and persistent heterogeneity across forecasters in the term structure of uncertainty.*

Individual forecasters differ in both their level and term structure of uncertainty. Figure 4 displays the variability of variance across forecasters over time for inflation. We find that heterogeneity of forecast uncertainty is both substantial and time-varying. See Figures A3 and A4 for analogous results for growth and unemployment. Notice, for each variable and horizon, that while uncertainty is higher during and after the recession, this increase comes primarily from the top half of the distribution. The 10th and 25th percentiles of uncertainty and even the median change relatively little, while the 75th and 90th percentiles increase more dramatically, pointing to the asymmetry in the evolution of cross-sectional distribution of individual uncertainty.

Moreover, for most respondents, 5-year uncertainty is greater than 1-year uncertainty, but a sizeable minority have an inverted term structure: 28% for inflation, 24% for growth, and 15% for unemployment. Other evidence of heterogeneity in the term structure of uncertainty comes from estimating the linearity regressions, as in Table 1, for each individual forecaster (for the 40 forecasters who provided at least 25 forecasts for each variable and horizon). In Table 5, we report summary statistics of the regression coefficients across forecasters. There is considerable variation across forecasters in how the longer-run uncertainty depends on shorter-run uncertainty, as shown by large standard deviations.

The level and term structure of uncertainty are persistent for individual forecasters (see Table A4). Also note that both the mean level and the mean slope are correlated across variables. That is, a forecaster with relatively high average inflation uncertainty also has relatively high average growth and unemployment uncertainty. And a forecaster with a relatively wide term structure of uncertainty for inflation is also likely to have a relatively wide term structure for growth and unemployment (Table A5).

11. Interestingly, Engelberg, Manski, and Williams (2009) find that U.S. SPF forecasters tend to report point predictions that give a more favorable view of the economy than do their subjective means/medians/modes from density forecasts.

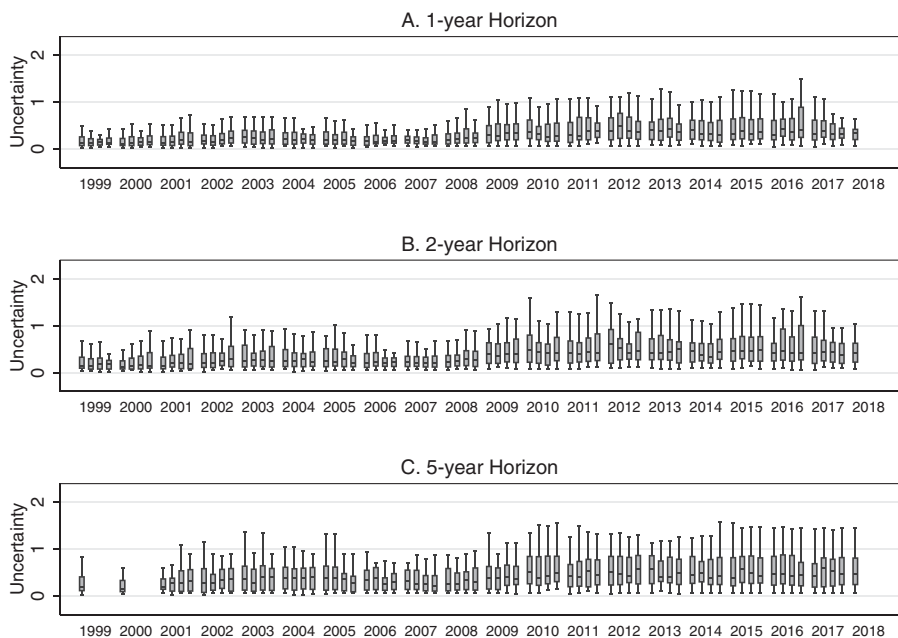


Fig 4. Distribution of Inflation Uncertainty across Forecasters over Time.

NOTES: ECB SPF data. Interior line is the median, bottom and top of boxes are the 25th and 75th percentiles, and bottom and top of lines are 10th and 90th percentiles of inflation uncertainty.

TABLE 5

FORECASTER HETEROGENEITY IN TERM STRUCTURE OF UNCERTAINTY

Variable	Level		Slope	
	mean	std. dev.	mean	std. dev.
Inflation	0.816	0.264	0.637	0.358
Growth	0.740	0.348	0.571	0.393
Unemployment	0.739	0.408	0.739	0.431

NOTES: The table shows the mean and standard deviation of the regression coefficient estimates from a regression of 5-year log uncertainty on 1-year log uncertainty (level) and the difference between 1- and 2-year log uncertainty (slope) estimated for each individual forecasters, using the 40 forecasters who make at least 25 density forecasts for each variable at each horizon.

### 3. A MODEL OF EXPECTATION UPDATING

We adopt the framework of Baker, McElroy, and Sheng (2020) that features Kalman filter updating and state-dependent information processing. We generalize their framework by allowing for time-varying uncertainty in both the signal and the noise and by studying multistep ahead forecasts and the associated uncertainty.

### 3.1 Information Environment and Kalman Filter Updating

We set forth a framework that represents the internal oracular mechanism of an agent, by which the observations are perceived by the agent as being composed of latent signal and noise processes, each of whose parameters are agent-dependent—but are known to the agent. The signal extraction machinery—including process models, parameters, and extraction algorithms—are internal to the agent, whereas the data inputs are external. Specifically, we suppose there are  $N$  agents tasked with forecasting several periods ahead an  $m$ -dimensional stationary Markov signal process  $\{\pi_t\}$ , which is perceived as coming through a publicly available channel that involves obfuscating noise; the presence of noise is essential to explaining the last three stylized facts, as we show below. Here the time index  $t$  is arbitrary, and would correspond to quarterly indexing for the data considered in this paper. Each agent is attempting to produce MSE optimal linear forecasts based on their own perception (or specification) of the dynamics of the signal and noise processes; our framework is designed to be a convenient proxy for their actual behavior. Thus, we suppose that the observation process for agent  $i$  is

$$y_t(i) = B(i)\pi_t + \eta_t(i), \quad (1)$$

where  $\{\eta_t(i)\}$  represents the observation noise for the  $i$ th agent, and the matrix  $B(i)$  corresponds to the manifestation of the signal. As (1) represents the internal oracular mechanism of agent  $i$ , the noise  $\{\eta_t(i)\}$  and the observation matrix  $B(i)$  can vary with  $i$ . In essence, the agent-specific signal is  $B(i)\pi_t$ , but taking  $B(i)$  equal to the identity matrix means that the signal  $\{\pi_t\}$  is common to all agents; in this case the agents may disagree about how the signal is perceived (through their observation noise  $\eta_t(i)$ ), but they do agree as to how the signal is defined (namely, as  $\pi_t$ ). This assumption allows us to isolate the impact of observation noise on interagent disagreement. Furthermore, we assume that  $\{\eta_t(i)\}$  is serially uncorrelated, has a time-varying covariance matrix  $\Sigma_t^\eta(i)$ , and is independent across agents.

Given that the agent seeks to generate an MSE optimal linear forecast, it is natural to express updates to previous forecasts through the Kalman filter—this will allow us to fluidly develop formulas expressing the term structure of uncertainty. We first formulate each forecaster’s Kalman filtering process, taking the variances  $\Sigma_t^\eta(i)$ , the parameters governing  $\{\pi_t\}$ , and the matrix  $B(i)$  as known—since these quantities are merely proxies for the agent’s internal assessment of the market. In other words, their internal framework includes a specification of both the models and parameters—including the time-varying variances—but not the realizations of the signal and noise processes. (In the end of this subsection, we discuss the impact of shocks to signal and noise.)

Suppose there exists a matrix  $G$  such that  $\pi_t = Gx_t$ , where  $x_t$  is a heteroskedastic Markovian state vector with transition equation given by

$$x_t = \Phi x_{t-1} + \epsilon_t \quad (2)$$

for  $t \geq 1$ , with initial value  $x_0$ . We assume the transition matrix  $\Phi$  has eigenvalues less than one in magnitude (this ensures the signal is stationary) and that the signal innovations  $\{\epsilon_t\}$  are uncorrelated with  $x_0$ , so that  $\epsilon_t$  is uncorrelated with  $x_{t-1}$  for  $t \geq 1$ . Each innovation's covariance matrix is denoted  $\Sigma_t^\epsilon$ . Let  $H(i) = B(i)G$ , so that combining (1) with  $\pi_t = Gx_t$  yields the observation equation

$$y_t(i) = H(i)x_t + \eta_t(i). \quad (3)$$

Equations (3) and (2) describe the information structure in state space form. We propose a flexible heteroskedastic VAR( $p$ ) class for the signal, where  $p$  is taken sufficiently large to approximate a generic signal. We can give  $\{\pi_t\}$  this structure if we set  $x'_t = [\pi'_t, \pi'_{t-1}, \dots, \pi'_{t-p+1}]$  with  $G = [I_m, 0, \dots]$  (and  $I_m$  is the  $m$ -dimensional identity matrix), and define the transition matrix by

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_p \\ I_m & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & I_m & 0 \end{bmatrix}.$$

If the agent considers only their own past information, then the  $h$ -step ahead forecast of the state vector is defined (for  $h \geq 1$ ) via

$$\hat{x}_{t+h|t}(i) = \mathbb{E}[x_{t+h}|y_1(i), \dots, y_t(i)],$$

and its mean square error matrix is  $P_{t+h|t}(i) = \text{Cov}[x_{t+h} - \hat{x}_{t+h|t}(i)]$ . (Note that the dependence on agent  $i$  enters not only through the agent-dependent forecast  $\hat{x}_{t+h|t}(i)$ , but also through models and parameters of  $x_t$  that are agent-dependent.) The Kalman filter updating for  $1 \leq t \leq T$  is standard in the literature and, for brevity, is omitted here; however, we note that the Kalman gain  $K_t(i)$  depends on  $i$  through the observation matrix  $B(i)$ . In order to obtain  $h$ -step ahead forecasts ( $h \geq 1$ ), we compute

$$\hat{\pi}_{t+1|t+1-h}(i) = G\Phi^{h-1}\hat{x}_{t+2-h|t+1-h}(i), \quad (4)$$

$$\text{Var}[\hat{\pi}_{t+1|t+1-h}(i) - \pi_{t+1}] = GP_{t+1|t+1-h}(i)G', \quad (5)$$

and the calculation of  $P_{t+1|t+1-h}(i)$  is further described below. So (4) provides an optimal linear estimate of the forecasted signal, given the presence of noise with time-varying variability.

Because the transition matrix  $\Phi$  is assumed to have eigenvalues less than one in magnitude, one should parameterize the VAR( $p$ ) matrix polynomial such that stability is guaranteed, as described in Roy, McElroy, and Linton (2019). Both the signal and noise covariance matrices  $\Sigma_t^\epsilon$  and  $\Sigma_t^\eta$  are parameterized as stochastic processes



taking values in the space of symmetric positive definite matrices. For each  $\Sigma_t$ , consider the Cholesky decomposition  $\Sigma_t = A_t \Omega_t A_t'$ , where  $A_t$  is unit lower triangular and  $\Omega_t$  is diagonal. The diagonal entries of  $\Omega_t$  each follow an exponential random walk, and  $A_t = \exp\{C_t\}$  (i.e.,  $A_t$  is the matrix exponential of  $C_t$ ), where  $C_t$  is a lower triangular with zeros on the diagonal. Each lower triangular element of  $C_t$  follows an independent random walk. This framework can be tailored to the user's specification through the parameter settings, which determine the dispersion for the random walk increments in the covariance matrix process.

A shock to the variability corresponds to a salient shift in the time-varying variance, and can be modeled by increasing the value by using a multiplicative factor (merely increasing the random walk innovation variances will not generate this behavior). In particular, a temporary shock to the variance at some time index  $\tau$  can be generated by scaling the entries of a single  $\Sigma_t$  by some  $a > 0$ , but without altering  $A_t$  or  $\Omega_t$ , so that the effect is transitory:

$$\Sigma_t = A_t \Omega_t A_t' \cdot (1 + a 1_{\{t=\tau\}}). \quad (6)$$

This ensures that  $\Sigma_\tau$  has values multiplied by  $1 + a$ , and is denoted as a temporary shock; this is like an additive outlier modifying a stochastic process.

A permanent shock at some time index  $\tau$  involves dilating  $\Sigma_t$  in the same manner as the temporary shock, but with the effect lasting for all times  $t \geq \tau$ :

$$\Sigma_t = A_t \Omega_t A_t' \cdot (1 + a 1_{\{t \geq \tau\}}). \quad (7)$$

This is like a level shift outlier that modifies a stochastic process. Whereas a temporary shock represents a transient aberration in the process, a permanent shock corresponds to a new state of affairs. Shocks to the variability in the signal process pertain to perceived changes in the true state of the market, with lower values of  $\Sigma_t^\xi$  corresponding to greater stability. Shocks to the variability in the noise process instead correspond with reduced confidence in the agent's assessment of the market's true state, as larger values of  $\Sigma_t^\eta$  indicate that the signal is buried under greater obscurity. Hence, lower values of  $\Sigma_t^\eta$  correspond to a greater confidence in the agent's ability to infer the latent signal.

### 3.2 Multistep-Ahead Forecasting

The three key ingredients of our framework are: (i) multistep-ahead forecasting, (ii) heteroskedastic signal and noise with shocks, and (iii) Kalman filter updating. The  $i$ th agent ( $1 \leq i \leq N$ ) observes the public data and makes the forecast through a signal extraction process. Given the data  $\{y_t(i)\}$  for  $1 \leq t \leq T$ , for each agent  $i$ , we obtain the  $h$ -step ahead forecast  $\hat{\pi}_{t+1|t+1-h}(i)$  from (4); calculation of the uncertainty relies upon  $P_{t+1|t+1-h}(i)$  in (5), which is a special case of the multistep ahead error covariance

$$R_{k,\ell}^{(ij)}(t) = \text{Cov}[\hat{x}_{t+1|t-k}(i) - x_{t+1}, \hat{x}_{t+1|t-\ell}(j) - x_{t+1}],$$

that is, the covariance of forecast errors for the  $i$ th agent ( $k + 1$  steps ahead) and the  $j$ th agent ( $\ell + 1$  steps ahead). The following result provides a recursive algorithm for computing these covariances, based upon recursions for the one-step ahead error covariances:

$$Q_{t+1|t}^{(ij)} = \text{Cov}[\widehat{x}_{t+1|t}(i) - x_{t+1}, \widehat{x}_{t+1|t}(j) - x_{t+1}]$$

between agents  $i$  and  $j$ . Note that setting  $j = i$  and  $\ell = k = h - 1$ , we obtain  $P_{t+1|t+1-h}(i) = R_{h-1, h-1}^{(ii)}(t)$ , and furthermore  $Q_{t+1|t}^{(ii)} = P_{t+1|t}(i)$ .

**PROPOSITION 1.** *The covariance of one-step ahead prediction errors across agents,  $Q_{t+1|t}^{(ij)}$ , can be computed recursively by*

$$\begin{aligned} Q_{t+1|t}^{(ij)} &= [\Phi - K_t(i)H(i)]Q_{t|t-1}^{(ij)}[\Phi - K_t(j)H(j)]' + \Sigma_t^\epsilon \\ &\quad + 1_{\{i=j\}}K_t(i)\Sigma_t^\eta(i)K_t(j)', \end{aligned} \quad (8)$$

with the initialization  $Q_{1|0}^{(ij)} = \text{Var}[x_1]$  for all  $i$  and  $j$ . The covariance of prediction errors across forecast horizons and agents,  $R_{k,\ell}^{(ij)}(t)$ , can be computed if  $k \leq \ell$  by

$$\begin{aligned} R_{\ell,\ell}^{(ij)}(t) &= \Phi^\ell Q_{t+1-\ell|t-\ell}^{(ij)} \Phi'^\ell + \sum_{n=0}^{\ell-1} \Phi^n \Sigma_{t+1-n}^\epsilon \Phi'^n \\ R_{\ell-1,\ell}^{(ij)}(t) &= \Phi^{\ell-1} (\Phi - K_{t+1-\ell}(i)H(i)) Q_{t+1-\ell|t-\ell}^{(ij)} \Phi'^\ell + \sum_{n=0}^{\ell-1} \Phi^n \Sigma_{t+1-n}^\epsilon \Phi'^n \\ R_{k,\ell}^{(ij)}(t) &= \Phi^k \prod_{n=k}^{\ell-1} (\Phi - K_{t-n}(i)H(i)) Q_{t+1-\ell|t-\ell}^{(ij)} \Phi'^\ell \\ &\quad + \sum_{m=2}^{\ell-k} \Phi^k \prod_{n=k}^{\ell-m} (\Phi - K_{t-n}(i)H(i)) \Sigma_t^\epsilon \Phi'^{\ell-m+1} + \sum_{n=0}^k \Phi^n \Sigma_{t+1-n}^\epsilon \Phi'^n, \end{aligned}$$

where  $k \leq \ell - 2$  in the last case, and where the matrix products are computed with the lowest index matrix first, and multiplying on the right by matrices of higher index.

Whereas (8) of Proposition 1 was proved in Baker, McElroy, and Sheng (2020) for the case of a common public noise, it is extended here to the case where the noise process can be different for the various agents—note that the last term is present only if  $i = j$ , being zero otherwise.

If we have interest in some linear composite of economic agents' results, say

$$\bar{\pi}_{t+h|t} = \sum_{i=1}^N w_i \widehat{\pi}_{t+h|t}(i) \quad (9)$$

for given weights  $w_i$ , then the corresponding target is  $\sum_{i=1}^N w_i \pi_{t+h}$ , which equals  $\pi_{t+h}$  when the weights sum to one. We call (9) the composite forecast.

Next, let the aggregate forecast uncertainty (of the state vector) be defined as

$$U_{t+h|t} = \sum_{i=1}^N w_i P_{t+h|t}(i),$$

which represents an average (across agents) of the variability in  $h$ -step ahead forecasting of the state vector. We can derive an expression for forecast uncertainty that is recursive in  $h$ , and reveals the term structure of uncertainty.

**PROPOSITION 2.** *The covariance of prediction errors across multiple agents at one forecast horizon can be recursively computed via*

$$R_{k+h,k+h}^{(ij)}(t+k+h) = \Phi^h R_{k,k}^{(ij)}(t+k) \Phi^h + \sum_{n=0}^{h-1} \Phi^n G' \Sigma_{t+1+k+h-n}^\epsilon G \Phi^n.$$

Hence the aggregate forecast uncertainty satisfies for  $h > 1$

$$U_{t+h|t} = \Phi^{h-1} U_{t+1|t} \Phi^{h-1} + \sum_{n=0}^{h-2} \Phi^n G' \Sigma_{t+h-n}^\epsilon G \Phi^n. \quad (10)$$

Proofs of Propositions 1 and 2 are in the online appendix. A key outcome of (10) is that the agent's forecast uncertainty depends on future values  $\Sigma_{t+h}^\epsilon$  of the signal's dynamics, which are subjectively determined by the agent—hence, future expectations about uncertainty are directly linked to present perceptions regarding the oracular mechanism's structure and parameters. Of course, these perceptions may be internalized, that is, formulated in a subconscious manner that precludes self-awareness of the forecasting procedure. Much of the term structure of uncertainty can be understood by examining the behavior of (10) for different values of the parameters—this is discussed below, through the device of simulations.

### 3.3 Stylized Facts Explained by the Model

Here we aim to explain the stylized facts of Section 3 through Propositions 1 and 2, and in particular via equation (10).

1. **Linearity of Term Structure of Uncertainty:** When the signal variability is fairly low (corresponding to a period of market stability), then the second term in (10) has more relative impact on  $U_{t+h|t}$ , and the term structure of uncertainty increases as a sum over powers of  $\Phi$ . Hence, when the eigenvalues of  $\Phi$  are close to unity—corresponding to a VAR( $p$ ) process that is close to nonstationarity—the term structure of uncertainty takes on an increasingly linear pattern. This is

most easily seen in a univariate AR(1) case, with homoskedastic signal, where (10) reduces to (taking  $N = 1$  for simplicity)

$$U_{t+h|t} = \Phi^{2(h-1)} U_{t+1|t} + \sum_{n=0}^{h-2} \Phi^{2n} \Sigma^\epsilon.$$

When SNR is high most of the MSE  $U_{t+1|t}$  is driven by signal variability  $\Sigma^\epsilon$ , so that the second summand dominates; in the limiting case that  $\Phi = 1$ , uncertainty is a linear function of  $h$  with slope  $\Sigma^\epsilon$  and intercept  $U_{t+1|t}$ . On the other hand, as the SNR drops (i.e., the noise variability is increasing) then  $U_{t+1|t}$  increases and the  $h$ -step ahead uncertainty is dominated by  $\Phi^{2(h-1)}$ , which is not a linear pattern for  $\Phi \approx 1$ .

2. The Difference between Long-run and Short-run Uncertainty is Procyclical: Although uncertainty generally increases with horizon, when the market is unstable the increase is sharper at short horizons. This effect is more prominent when the signal is perceived as being less persistent. When the noise variability is also high, there is a low SNR, which gives relatively more impact to  $U_{t+1|t}$  as is apparent from (10). On the other hand, for less persistent processes the matrix  $\Phi^{h-1}$  will quickly be small (in matrix norm) for larger values of  $h$ . Hence, in the low SNR regime, the net effect is a larger uncertainty at short forecast horizons relative to longer horizons *for less persistent signals*; when the signal is more persistent, the uncertainty is increased by a similar degree across horizons. Thus, the procyclical behavior is not apparent for highly persistent signals.
3. Forecasters are Overconfident at all Horizons: Overconfidence can be modeled as the agent having perceived a low noise variability; the state space framework of this section represents an agent's internal oracular mechanism, so they may specify quantities (such as  $\Sigma_t^j(i)$ ) as lower than is genuinely warranted by the market. This misperception (or misspecification) lowers their forecast uncertainty uniformly across horizons—as  $\Sigma_t^j$  only enters into the term  $U_{t+1|t}$  of formula (10) for  $U_{t+h|t}$ . If the noise variability were higher, the uncertainty would be higher at all horizons.
4. Heterogeneity across Forecasters in the Term Structure of Uncertainty: Some agents may have less uncertainty at longer horizons, and this arises in a situation where they believe that a current market instability condition will soon be rectified, and good times will return. We can model this as associating an unstable market behavior for short-term forecasts, and stable market behavior for long-term forecasts. That is, we let  $U_{t+h|t}$  be determined by larger values of  $\{\Sigma_t^\epsilon\}$  when  $h$  is small, and conversely when  $h$  is large we use smaller values of  $\{\Sigma_t^\epsilon\}$ .

The interplay between signal and noise that is metrized through SNR plays an essential role in establishing the last three stylized facts; whereas a basic VAR model

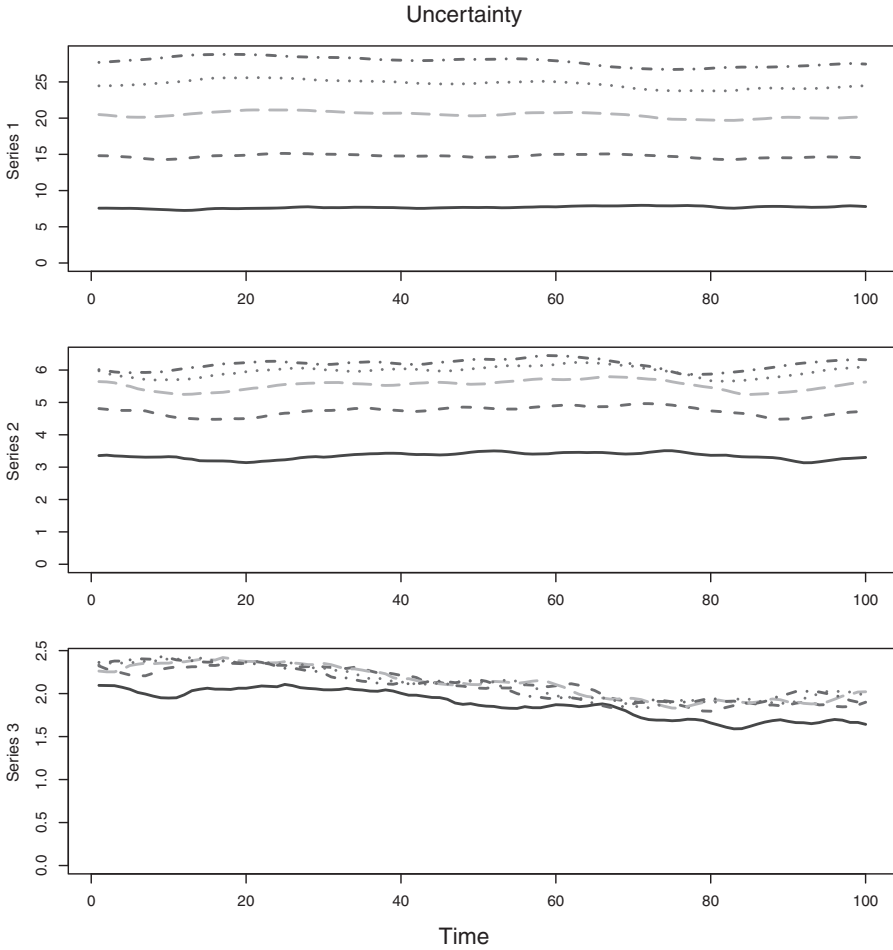


Fig 5. Simulated Term Structure of Uncertainty.

NOTES: Simulated term structure of uncertainty of a typical agent for a stable market regime (i.e., low variability in the signal process) with high signal-to-noise ratio.  $h = 4$  (solid),  $h = 8$  (dashed),  $h = 12$  (long dash),  $h = 16$  (dotted), and  $h = 20$  (dot-dash). Series 1 has high persistence (0.97), Series 2 has moderate persistence (0.90), and Series 3 has weaker persistence (0.80).

alone can explain the linearity of term structure uncertainty, it cannot account for the other empirically observed phenomena.

### 3.4 Simulation Evidence

We now provide simulation evidence for the above stylized facts, focusing on horizons  $h = 4, 8, 12, 16, 20$ , which correspond to 1-year ahead through 5-year ahead

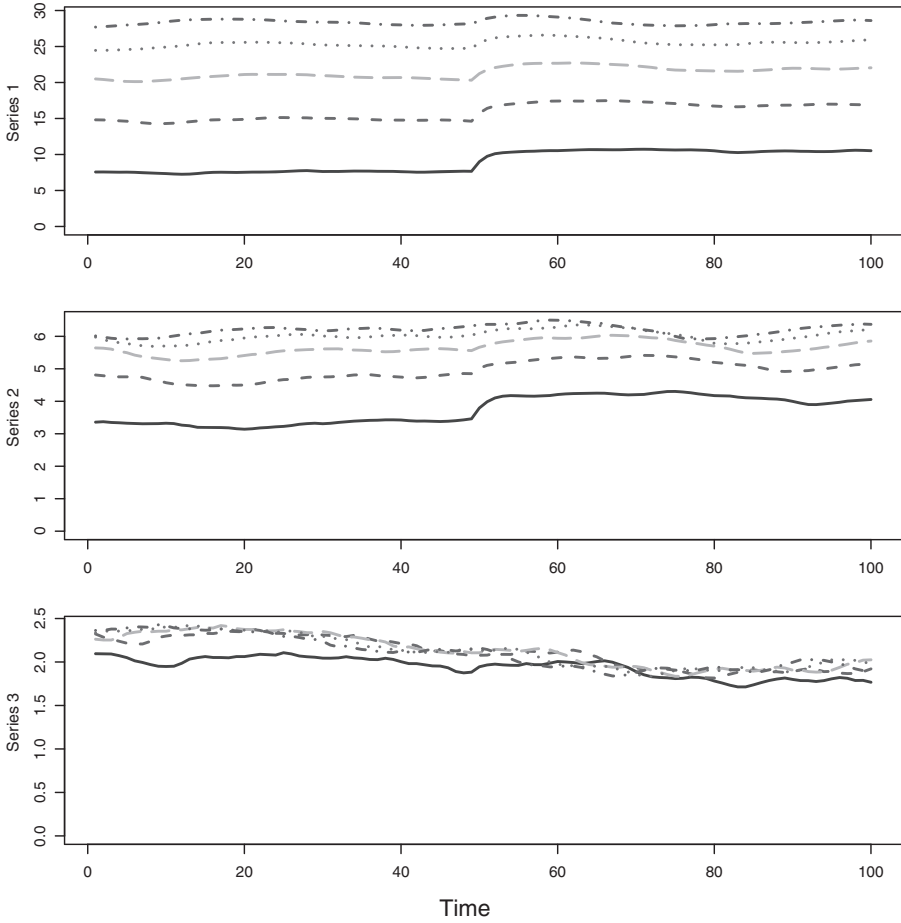


Fig 6. Simulated Term Structure of Uncertainty after a Shock.

NOTES: Simulated term structure of uncertainty of a typical agent for a stable market regime (i.e., low variability in the signal process), initially with a high signal-to-noise ratio that becomes lower half-way through the sample (at time  $t = 50$ ).  $h = 4$  (solid),  $h = 8$  (dashed),  $h = 12$  (long dash),  $h = 16$  (dotted), and  $h = 20$  (dot-dash). Series 1 has high persistence (0.97), Series 2 has moderate persistence (0.90), and Series 3 has weaker persistence (0.80).

forecasts for annual data. For purposes of illustration, we consider a trivariate ( $m = 3$ ) VAR(1) with coefficient matrix given by

$$\Phi_1 = \begin{bmatrix} 0.97 & 0.10 & 0.20 \\ 0.00 & 0.90 & -0.20 \\ 0.00 & 0.00 & 0.80 \end{bmatrix}.$$

This matrix has eigenvalues 0.97, 0.90, and 0.80, and as a result the first series has the highest persistency, followed by the second, and the third series has the lowest

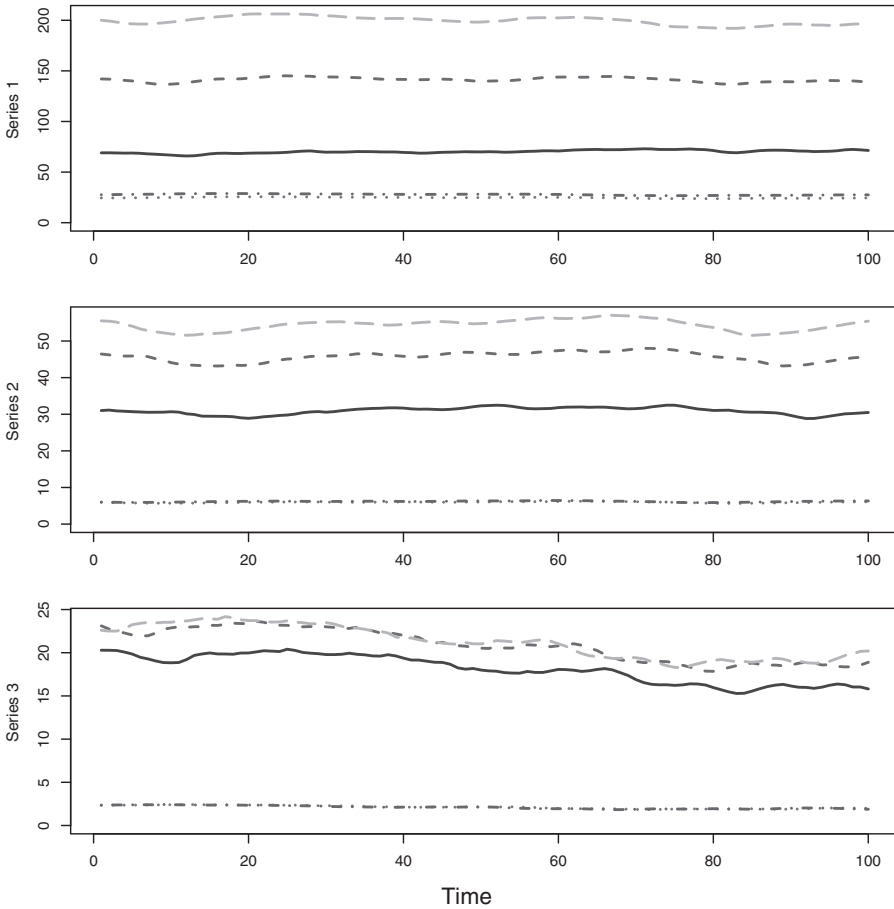


Fig 7. Simulated Heterogeneity in the Term Structure of Uncertainty.

NOTES: Simulated term structure of uncertainty of a typical agent when low horizons ( $h = 4, 8, 12$ ) correspond to an unstable market regime (i.e., high variability in the signal process), but high horizons ( $h = 16, 20$ ) correspond to a stable market regime (i.e., low variability in the signal process).  $h = 4$  (solid),  $h = 8$  (dashed),  $h = 12$  (long dash),  $h = 16$  (dotted), and  $h = 20$  (dot-dash). Series 1 has high persistence (0.97), Series 2 has moderate persistence (0.90), and Series 3 has weaker persistence (0.80).

persistence. The parameters of the data generating process are chosen for the purposes of illustration, and so as to represent three series with differing levels of persistency—in rough correspondence with unemployment (which is most persistent), followed by inflation and GDP growth. We take  $B(i)$  to be the identity matrix.

The stable market regime corresponds to setting the innovation variance to 0.1 for the Gaussian random variables driving both the exponential random walk  $\Omega_t$  and the matrix exponential of  $C_t$  for the signal—the matrices  $\Sigma_t^\epsilon$  are stochastically generated.

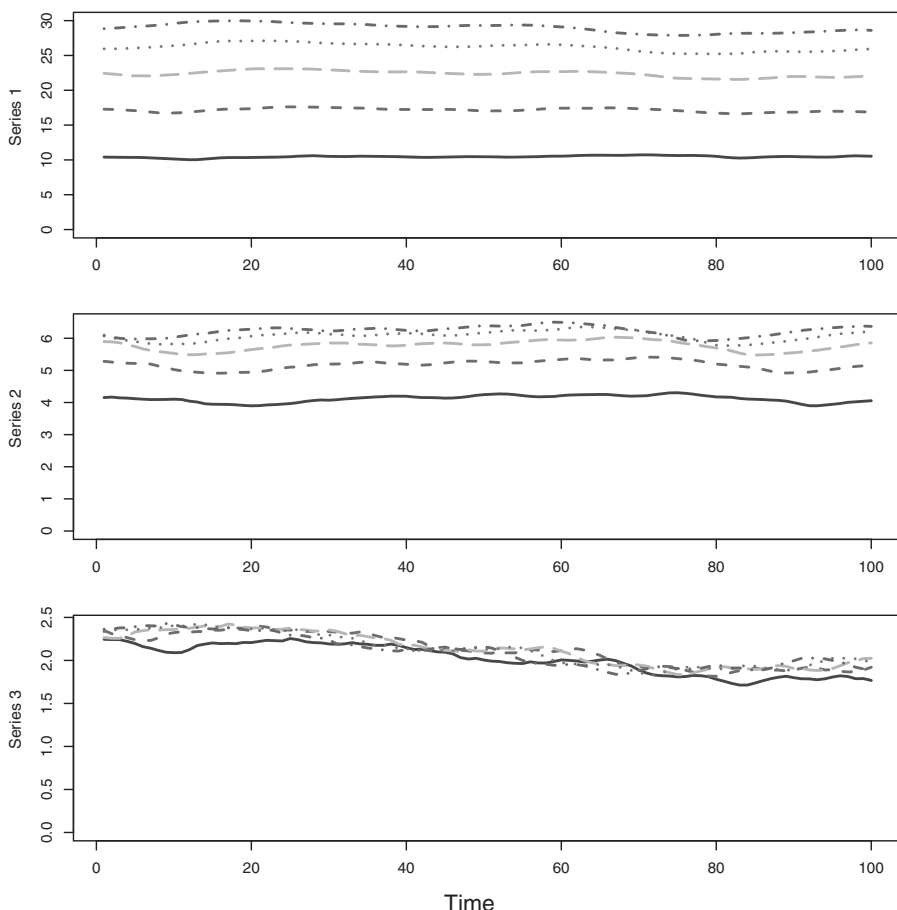


Fig 8. Overconfidence and Perceived Uncertainty: Simulation Evidence.

NOTES: Term structure of uncertainty of a typical agent for a stable market regime (i.e., low variability in the signal process) with low signal-to-noise ratio.  $h = 4$  (solid),  $h = 8$  (dashed),  $h = 12$  (long dash),  $h = 16$  (dotted), and  $h = 20$  (dot-dash). Series 1 has high persistence (0.97), Series 2 has moderate persistence (0.90), and Series 3 has weaker persistence (0.80).

An unstable market regime is generated by increasing each  $\Sigma_t^\epsilon$ ; this can be accomplished by the artifice of introducing a sustained signal shock at time  $t = 0$ , according to the methods described in (6) and (7) with  $a = 9$ . The SNR is taken to be high by default, which is accomplished by setting the noise innovation variance to 0.01. The signal and noise processes were generated to length  $T = 100$  with a burn-in period of 500, and possibly with shocks according to the stylized facts we seek to illustrate.

We begin by demonstrating linearity: Figure 5 corresponds to a stable market regime with a high SNR. The horizons are  $h = 4, 8, 12, 16, 20$ , which are given colors and line types corresponding to those of Section 3 (the cases  $h = 4, 8, 20$ ). Here



the linear structure is visibly evident in the upper panel, where the persistency is strongest, and to a lesser extent in the middle panel. For the lower panel, the persistency is weak enough that there is no linear term structure. As the SNR is lowered (i.e., noise variability decreases) the linear pattern begins to break down.

Next, we consider long-run versus short-run behavior, and the procyclical tendencies. Consider Figure 6, which focuses on a stable market regime. Initially there is a high SNR, but at time point 50 we introduce a sustained shock to the noise variability, which lowers the SNR at subsequent times. The general effect is an increase to forecast uncertainty, but note the difference between the upper and lower panels: in the upper panel (highly persistent signal) uncertainty is affected similarly at all horizons, whereas in the lower panel (less persistent signal) the long-horizon uncertainty receives little impact while the shorter horizons ( $h = 4, 8$ ) are increased. A similar behavior occurs in an unstable market regime.

For the heterogeneity of forecasters, we set the signal variability to be higher (essentially through the mechanism of a permanent shock at time  $t = 0$ ) for horizons  $h = 4, 8, 12$ , but use the lower value for signal variability when  $h = 16, 20$ . In Figure 7, we see that the regular ordering of uncertainty has been altered in each panel; now the higher horizon uncertainty is actually lower than that of the lower horizon forecasts.

We claimed above that overconfidence can be modeled through an artificially low noise variability. Looking back at Figure 5, we see the uncertainty pattern for a stable market regime with high SNR (or a low noise variability). Suppose that this corresponds to an artificially low noise variability, due to overconfidence; this is a perceived uncertainty. Furthermore, suppose that the true market conditions indicate a higher degree of noise variability (or lower SNR)—such a case is displayed in Figure 8. In comparing the uncertainties for Figure 5 with those of Figure 8, we see that (for all the panels) those of the former case are artificially low, reflecting forecaster hubris.

#### 4. CONCLUSION

It is well known that uncertainty rises in recessions and crises—it did so in the Great Recession, and is poised to do so again in 2020. Less known is that the term structure of uncertainty also changes in recessions, becoming much narrower. We have constructed measures of individual forecasters' subjective uncertainty at horizons ranging from 1 to 5 years, using density forecasts from the ECB SPF, and documented four stylized facts. First, the term structure of uncertainty is linear—uncertainty at the 1- and 2-year horizons can almost perfectly predict uncertainty at the 5-year horizon. Second, the slope of the term structure of uncertainty is procyclical. Third, forecasters are overconfident at all horizons in their forecasts for key macro variables, for example, output growth, inflation, and unemployment. Fourth, forecasters substantially

differ from each other in their term structure of uncertainty, and this heterogeneity is persistent.

Guided by our stylized facts, we have built a model of expectations formation process under an information structure with private and public channels of information. Our model is similar to Baker, McElroy, and Sheng (2020), which features Kalman filter updating and state-dependent information processing. They assume heteroskedasticity in the public signal and study the one-period ahead point forecast only. In contrast, we allow for time-varying uncertainty in both the signal and the noise. When the perceived noise variability is substantially lower than the actual noise variability, agents lower their forecast uncertainty uniformly across horizons, resulting in overconfidence. Most importantly, we generalize their framework by deriving the evolution of forecasts and the associated uncertainty across multiple horizons.

Our theory highlights the role of perceived persistence in understanding multistep ahead expectations formation, since the linearity and procyclical slope of the term structure cannot be explained by models of sticky information, classic noisy information, or simple VAR. When agents perceive the signal as being very persistent, the term structure of uncertainty takes a linear pattern, and following shocks, the uncertainty is increased by a similar degree across horizons. For less persistent signals, however, the uncertainty increases more at shorter horizons relative to longer horizons, leading to a procyclical term structure. Future research is warranted in exploring whether this perceived persistence in forming expectations, in addition to habit formation and adjustment costs, is another source of macro-economic persistence.

## APPENDIX A

TABLE A1  
VARIABLES FORECASTED BY THE EUROPEAN CENTRAL BANK SURVEY OF PROFESSIONAL FORECASTERS

Name	Notation	Description
Inflation	$\pi$	Year on year percentage change of the Harmonised Index of Consumer Prices (HICP) published by Eurostat
Growth	$g$	Year on year percentage change of real GDP based on ESA definition
Unemployment	$u$	Unemployment as percentage of labor force based on Eurostat definition

NOTES: For more information, see "ECB Survey of Professional Forecasters (SPF): Description of SPF Dataset."

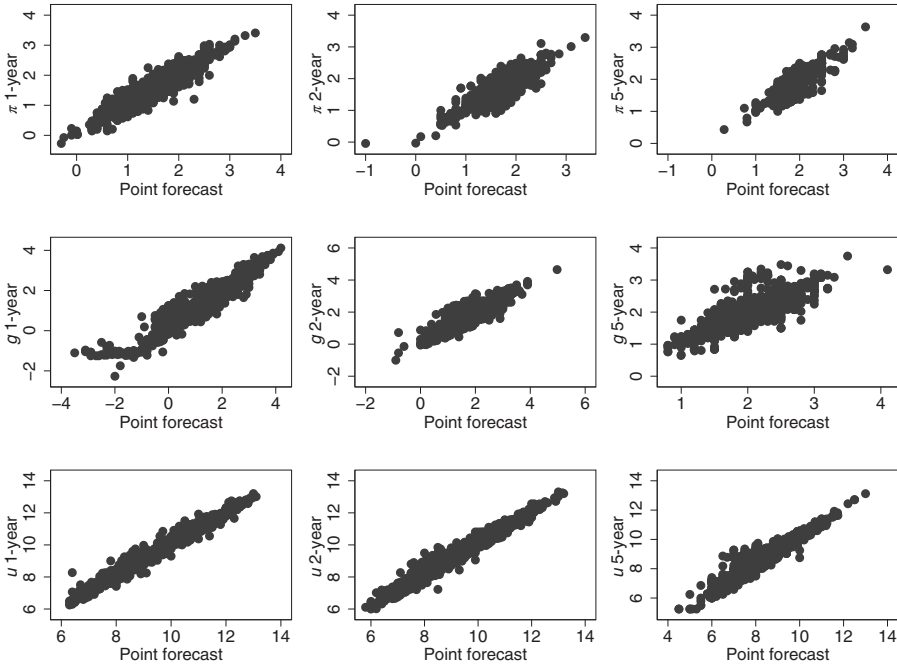


Fig A1. Comparison of Point Forecasts and Density Forecast Means.

NOTES: ECB SPF data, pooled across forecast dates. Density mean is estimated by maximum likelihood using generalized beta distribution.  $\pi$  – inflation,  $g$  – growth, and  $u$  – unemployment.

TABLE A2  
SURVEY RETURN DATES AND AVAILABLE INFORMATION

Quarter	Return date	Last $\pi$ obs.	Last $g$ obs.	Last $u$ obs.
1	Late Jan/early Feb	Dec	Q3	Nov
2	Late April/early May	March	Q4	Feb
3	Late July/early Aug	June	Q1	May
4	Late Oct/early Nov	Sep	Q2	Aug

NOTES: For more information, see “ECB Survey of Professional Forecasters (SPF): Description of SPF Dataset.”

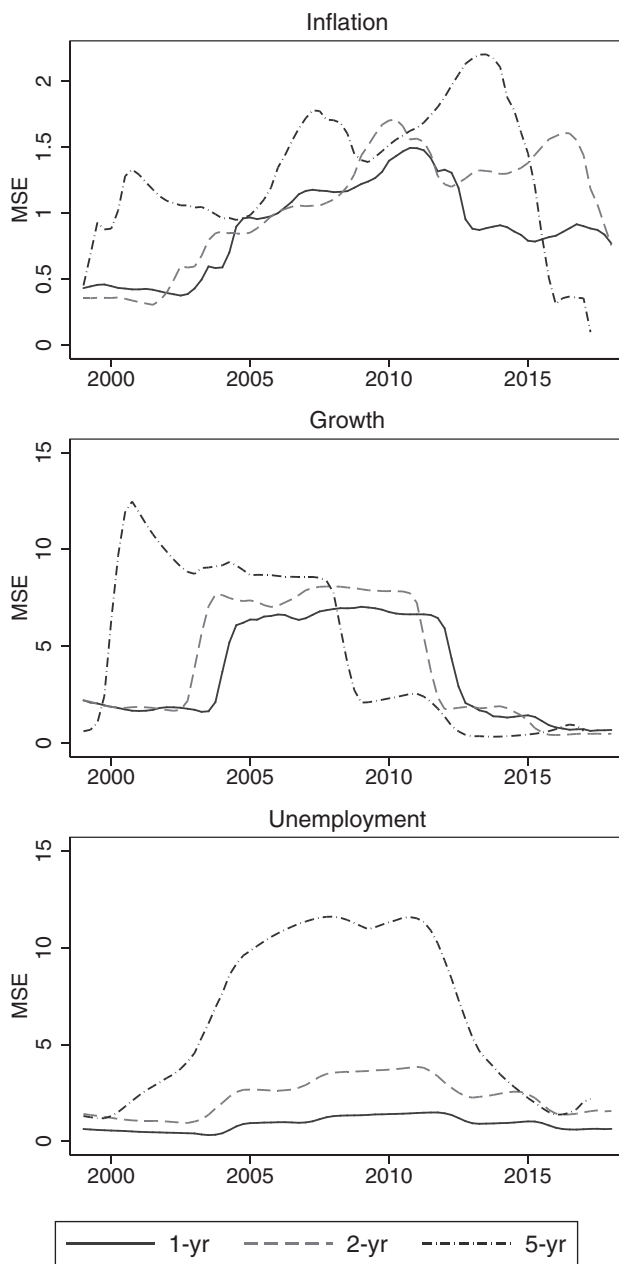


Fig A2. Time Series of Mean Squared Error.

NOTES: ECB SPF data. Mean of the squared difference between forecast made at time  $t$  and realization in  $t + 1, t + 2$ , or  $t + 5$ . Figure displays the centered 8-year moving average.

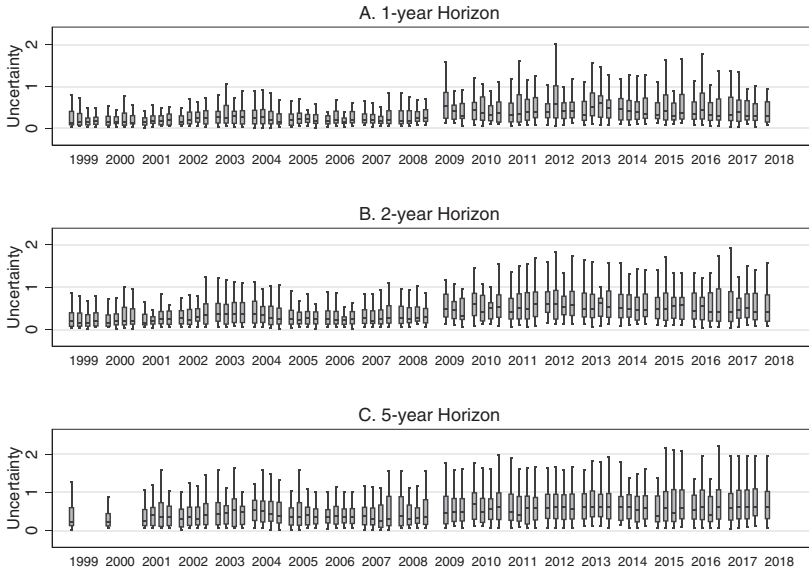


Fig A3. Distribution of Growth Uncertainty across Forecasters over Time.

NOTES: ECB SPF data. Interior line is the median, bottom and top of boxes are the 25th and 75th percentiles, and bottom and top of lines are 10th and 90th percentiles of growth uncertainty.

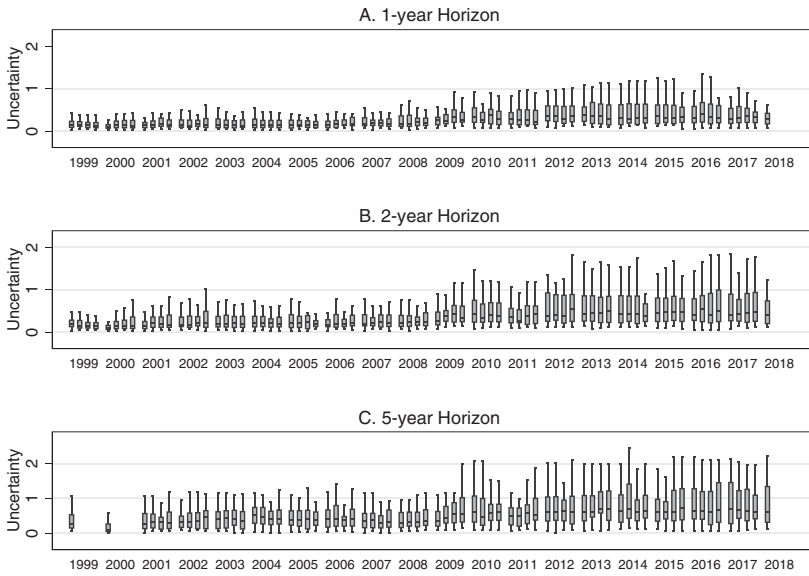


Fig A4. Distribution of Unemployment Uncertainty across Forecasters over Time.

NOTES: ECB SPF data. Interior line is the median, bottom and top of boxes are the 25th and 75th percentiles, and bottom and top of lines are 10th and 90th percentiles of unemployment uncertainty.

TABLE A3  
LINEARITY OF THE TERM STRUCTURE OF AGGREGATE UNCERTAINTY PRE- AND POST-2008

	(1) Inflation	(2) Inflation	(3) Growth	(4) Growth	(5) Unemp	(6) Unemp
1-year	0.85*** (0.10)	0.85*** (0.06)	0.64*** (0.11)	0.73*** (0.06)	1.03*** (0.21)	0.86*** (0.07)
2-yr minus 1-yr	1.00*** (0.18)	0.95*** (0.28)	0.30** (0.14)	0.59*** (0.19)	1.63*** (0.28)	1.09*** (0.21)
Constant	-0.09 (0.19)	-0.11 (0.08)	-0.21 (0.18)	-0.11 (0.08)	0.34 (0.39)	0.11 (0.11)
Observations	30	41	30	40	30	41
R <sup>2</sup>	0.76	0.78	0.50	0.67	0.74	0.82
Sample	Pre	Post	Pre	Post	Pre	Post

NOTES: Robust, time-clustered standard errors in parentheses. Dependent variable is log uncertainty at 5-year horizon for indicated variable. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ . In columns (1), (3), and (5), time sample is 1999–2007. In columns (2), (4), and (6), time sample is 2008–18.

TABLE A4  
PERSISTENCE OF UNCERTAINTY AND TERM STRUCTURE

	Inflation		Growth		Unemployment	
	(1) logU	(2) IncrTerm	(3) logU	(4) IncrTerm	(5) logU	(6) IncrTerm
L.logU	0.87*** (0.01)		0.81*** (0.02)		0.86*** (0.02)	
L.IncrTerm		0.40*** (0.04)		0.23*** (0.04)		0.41*** (0.05)
Constant	-0.19*** (0.03)	0.44*** (0.04)	-0.25*** (0.03)	0.58*** (0.04)	-0.20*** (0.04)	0.50*** (0.05)
Observations	2,826	1,996	2,725	1,913	2,537	1,750
R <sup>2</sup>	0.76	0.23	0.68	0.10	0.74	0.19

NOTES: Robust standard errors in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ . In columns (1), (3), and (5), dependent variable is log uncertainty at the one-year horizon, for inflation, growth, and unemployment, respectively. In columns (2), (4), and (6), dependent variable is a dummy variable indicating that uncertainty is higher at the 5-year horizon than at the 1-year horizon, for inflation, growth, and unemployment, respectively. In each column, the independent variable is a one-quarter lag of the dependent variable.

TABLE A5  
CORRELATION OF AVERAGE LEVEL AND SLOPE OF TERM STRUCTURE ACROSS VARIABLES

	<i>Levels</i>	
$\ln U_{i1}^{\pi}$	$\ln U_{i1}^g$	$\ln U_{i1}^u$
$\ln U_{i1}^g$	0.96	0.88
	<i>Slopes</i>	
$\ln U_{i5}^{\pi} - \ln U_{i1}^{\pi}$	$\ln U_{i5}^g - \ln U_{i1}^g$	$\ln U_{i5}^u - \ln U_{i1}^u$
$\ln U_{i5}^g - \ln U_{i1}^g$	0.74	0.35
		0.27

NOTES: The table shows the correlation across variables of the mean level and slope of the term structure of uncertainty for the 40 forecasters who made at least 25 forecasts of each variable and horizon.

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